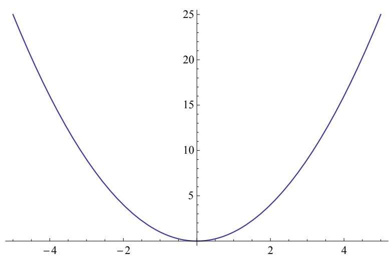
Business PreCalculus MATH 1643 Section 004, Spring 2014
Lesson 14: Properties of Functions

We will study the behavior of a function according to the changes of the variable. Let f be a function and let x and z be any two numbers in an open **interval** (a, b) **contained in the domain of** f.

Definition 1. **Increasing Function:** *The function f is called an* ***increasing function*** *on* ***(a,b)*** *if $x < z$ implies $f(x) < f(z)$.*

Definition 2. **Decreasing Function:** *The function f is called a* ***decreasing function*** *on* ***(a,b)*** *if $x < z$ implies $f(x) > f(z)$.*

Example 1. *The graph of the function $f(x) = x^2$ is given by*



which is increasing on the interval $(0, \infty)$ and decreasing on the interval $(-\infty, 0)$.

Definition 3. **Constant Function:** *The function f is called a* ***constant function*** *on* ***(a,b)*** *if $x < z$ implies $f(x) = f(z)$.*

Definition 4. **Even Function:** *A function f is called an* ***even function*** *if* ***for each x in the domain of f, $-x$ is also in the domain of f and*** *$f(-x) = f(x)$. Note that the graph of an even function is symmetric with respect to the y-axis.*

Example 2. *The function $f(x) = x^2$ is an even function because*

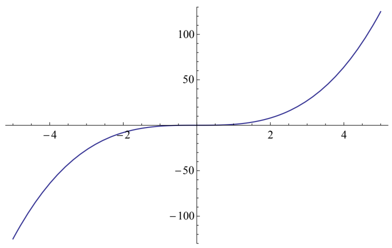
$$\begin{aligned} f(-x) &= (-x)^2 \qquad \text{Replace } x \text{ with } -x. \\ &= x^2 \\ &= f(x) \end{aligned}$$

Definition 5. **Odd Function:** *A function f is called an* ***odd function*** *if* ***for each x in the domain of f, $-x$ is also in the domain of f and*** *$f(-x) = -f(x)$. Note that the graph of an odd function is symmetric with respect to the origin.*

Example 3. *The function $f(x) = x^3$ is an odd function because*

$$\begin{aligned} f(-x) &= (-x)^3 \qquad \text{Replace } x \text{ with } -x. \\ &= -x^3 \\ &= -f(x) \end{aligned}$$

Its graph is given by



Definition 6. **Average Rate of Change of a Function:** *The average rate of change of $f(x)$ as x varies from a to b is defined by*

$$\frac{f(b) - f(a)}{b - a}$$

Example 4. *Find the average rate of change of $f(x) = 2 - 3x^2$ as x changes from 1 to 3.*

Solution:

$$f(1) = 2 - 3(1)^2 = -1$$
$$f(3) = 2 - 3(3)^2 = -25$$

Then

$$\frac{f(b) - f(a)}{b - a} = \frac{-25 - (-1)}{3 - 1} = -12.$$

Definition 7. **Difference Quotient** *For a function f, the difference quotient is defined by*

$$\frac{f(x + h) - f(x)}{h}$$

Example 5. *Let $f(x) = 2 - 3x^2$, find its difference quotient.*
Solution:

$$f(x) = 2 - 3x^2$$

$$\begin{aligned} f(x+h) &= 2 - 3(x+h)^2 \\ &= 2 - 3(x^2 + 2xh + h^2) \\ &= 2 - 3x^2 - 6xh - 3h^2 \end{aligned}$$

Then

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(2 - 3x^2 - 6xh - 3h^2) - (2 - 3x^2)}{h} \\ &= \frac{2 - 3x^2 - 6xh - 3h^2 - 2 + 3x^2}{h} \\ &= \frac{-6xh - 3h^2}{h} \\ &= \frac{h(-6x - 3h)}{h} \\ &= -6x - 3h \end{aligned}$$